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VELOCITY VECTOR FIELD FOR FLUID FLOW

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ABSTRACT

The description of a fluid flow requires a specification or determination of the velocity field, i.e. a specification of the fluid velocity at every point in the region. In general, this will define a vector fieldof position and time, $\mathbf{u} = \mathbf{u}(x, t)$.Steady flowoccurs when $\mathbf{u}$ is independent of time (i.e., $\partial\mathbf{u}/\partial t \equiv 0$).Otherwisethe flow is unsteady.Streamlinesare lines which at a given instant are everywhere in the directionof the velocity(analogous to electric or magnetic field lines). In steady flow thestreamlines are independent of time, but the velocity can vary in magnitude along astreamline (as in flow through a constriction in a pipe). Particle pathsare lines traced out by "marked" particles as time evolves. Insteady flow particle paths are identical to streamlines; in unsteady flow they aredifferent, and sometimes very different. Particle paths arevisualized in the laboratoryusing small floating particles of the same density as the fluid. Sometimes they arereferred to as trajectories.Filament lines or streaklines are traced out over time by all particles passingthrough a given point; they may be visualized, for example, using a hypodermicneedle andreleasing a slow stream of dye. In steady flow these are streamlines; inunsteady flow they are neither streamlines nor particle paths.It should be emphasized that streamlines represent the velocity field at a specific constant of time, whereas particle paths and streaklines provide a representation ofthe velocity field over a finite period of time. In the laboratory we can obtaina record of streamlines photographically by seeding the fluid with small neutrallybuoyant particles that move with the flow and taking a short exposure (e.g. 0.1 sec),long enough for each particle to trace out a short segment of line; the eye readilylinks these segments into continuous streamlines. Particle paths and streaklines areobtained from a time exposure long enough for the particle or dye trace to traversethe region of observation.

Keywords: Scalar fields, steady flow, mass continuity equation, Bernoulli's principle

Introduction:

When we describe the flow of a fluid like water, we may think of the movement ofindividual particles. These particles interact with each other through

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forces. We couldthen apply our laws of motion to each individual particle in the fluid but because thenumber of particles is very large,this would be an extremely difficult computationproblem. Instead we shall begin by mathematicallydescribing the state of moving fluidby specifying the velocity of the fluid at each point in space and at each instant in time.

For the moment we will choose Cartesian coordinates and refer to the coordinates of apoint in space by the ordered triple (x, y, z) and the variable t to describe the instant intime, but in principle we may chose any appropriate coordinate system appropriate fordescribing the motion. The distribution of fluid velocities is described by the vectorvalues function$v(x, y, z,t)$. This represents the velocity of the fluid at the point (x, y, z)at the instant t. The quantity$\mathbf{v}(x, y, z,t)$ is called the **velocity vector** field. It can bethought of at each instant in time as a collection of vectors, one for each point in spacewhose direction and magnitude describes the direction and magnitude of the velocity ofthe fluid at that point (Fig:1). This description of the velocity vector field of thefluid refers to fixed points in space and not to fixed moving particles in the fluid.

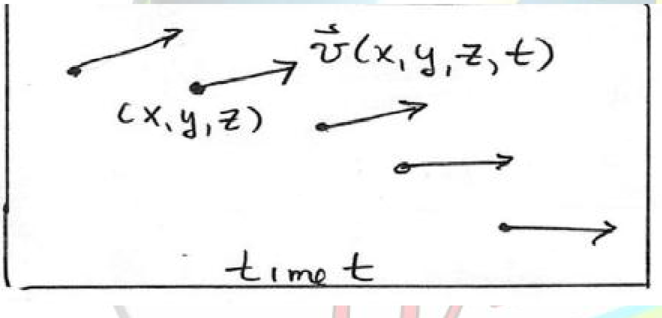

Fig:1 Velocity vector field for fluid flow at time *t*

We shall introduce functions for the pressure $P(x, y, z,t)$ and the density $\rho(x, y, z,t)$ ofthe fluid that describe the pressure and density of the fluid at each point in space and at Fig: 2each instant in time. These functions are called **scalar fields** because there is only onenumber with appropriate units associated with each point in space at each instant in time.In order to describe the velocity vector field completely we need three functions$v_x(x, y, z,t)$,$v_y(x, y, z,t)$, and $v_z(x, y, z,t)$.

For a non-ideal fluid, the differentialequations satisfied by these velocity component functions are quite complicated andbeyond the scope of this discussion. Instead, we shall primarily consider the special caseof **steady flow** of a fluid in which the velocity

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at each point in the fluid does not changein time. The velocities may still vary in space (non-uniform steady flow).

Let's trace the motion of particles in an ideal fluid undergoing steady flow during asuccession of intervals of duration dt .

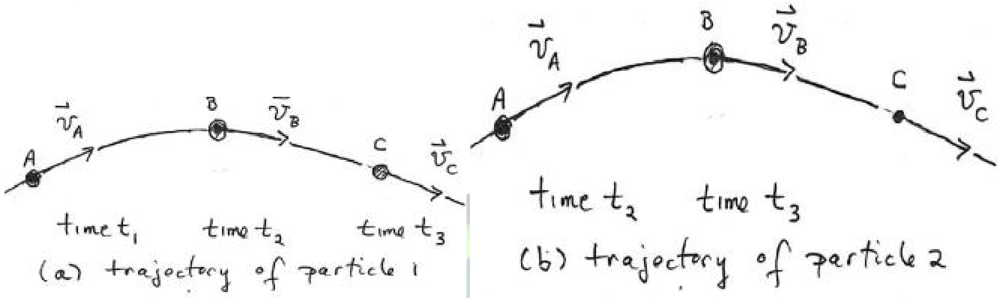

Fig:2(a) trajectory of particle 1, (b) trajectory of particle 2

Consider particle1 located at point A with coordinates(x_A, y_A, z_A).At the instantt_1 ,particle 1 will have velocity$\mathbf{v}(x_A, y_A, z_A)$and move to apoint $B$with coordinates(x_B, y_B, z_B),arriving there at the instant $t_2 = t_1 + dt$. During the next interval, particle 1will move to point C arriving there at instant $t_3 = t_2 + dt,$ where it has velocity$\mathbf{v}(x_B, y_B, z_B)$ (Fig:2(a)). Because the flow has been assumed to be steady, at instantt_2 , a different particle, particle 2, is now located at point A but it has the same velocity$\mathbf{v}(x_A, y_A, z_A)$particle 1 had at point A and hence will arrive at point B at the end ofthe next interval, at the instant $t_3 = t_2 + dt$ (Fig:2(b)). In the third interval, particle 2,which began the interval at point B will end the interval at point C. In this way every have the same velocities at points along a streamline because we have not assumed thatthe velocity field is uniform. Particle that lies on the trajectory that our first particle traces out in time will follow thesame trajectory. This trajectory is called a *streamline*. The particles in the fluid will not

Mass Continuity Equation: A set of streamlines for an ideal fluid undergoing steady flow in which there are nosources or sinks for the fluid is shown in Fig:3.

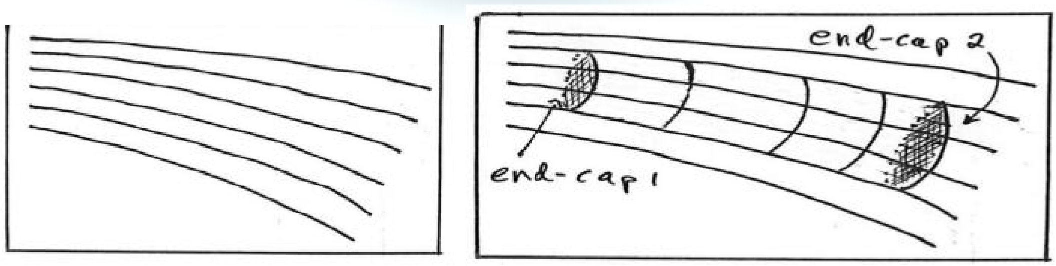

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Fig:3 Set of streamlines for anideal fluid flow**Fig:4** Flux Tube associated with set of streamlines

We add to the flow tube two open surface (end-caps 1 and 2) that are perpendicular tovelocity of the fluid, of areas A_1 and A_2 respectively. Because all fluid particles thatenter end-cap 1 must follow their respective streamlines, they must all leave end-cap 2. Ifour streamlines that form the tube aresufficiently close together, we can assume that thevelocity of the fluid in the vicinity of each end-cap surfaces is uniform.

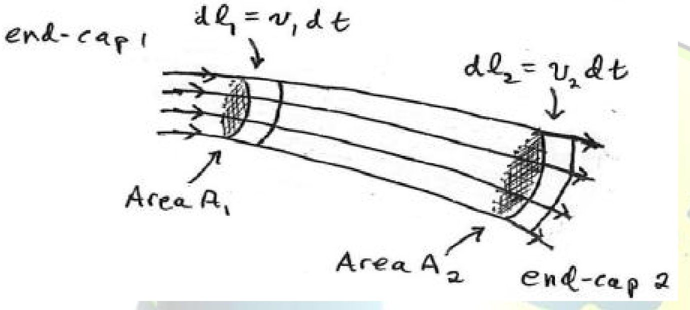

Fig: 5 Mass flow through flux tube

Let v_1 denote the speed of the fluid near end-cap 1 and v_2 denote the speed of the fluidnear end-cap 2. Let ρ_1 denote the density of the fluid near end-cap 1 and $\rho_2$denote thedensity of the fluid near end-cap 2. The amount of mass that enters and leaves the tube(Fig:4) in a time interval*dt*can be calculated as follows (Fig:5). Suppose we consider asmall volume of space of cross-sectional area A_1 and length $dl_1 = v_1 dt$ near end-cap 1.

The mass that enters the tube in time interval *dt*is

$$dm_1 = \rho_1 dV_1 = \rho_1 A_1 dl_1 = \rho_1 A_1 v_1 dt \ldots.. (1)$$

In a similar fashion, consider a small volume of space of cross-sectional area A_2 andlength $dl_2 = v_2 dt$ near end-cap 2

.

The mass that leaves the tube in the time interval *dt* isthen

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$$dm_2 = \rho_2 dV_2 = \rho_2 A_2 dl_2 = \rho_2 A_2 v_2 dt......(2)$$

An equal amount of mass that enters end-cap 1 in the time interval *dt* must leave end-cap2 in the same time interval, thus $dm_1 = dm_2$.Therefore usingEqs. (1) and (2),

we have that $\rho_1 A_1 v_1 dt = \rho_2 A_2 v_2 dt$.

Dividing through by *dt* implies that

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \text{(steady flow) ...... (3)}$$

Eq.(3) generalizes to any cross sectional area *A* of the thin tube, where the densityis ρ, and the speed is *v* ,

$$\rho A v = \text{constant (steady flow) ..... (4)}$$

Eq. (3) is referred to as the *mass continuity equation for steady flow*. If we assumethe fluid is incompressible, then Eq.(3) becomes

$$A_1 v_1 = A_2 v_2 \text{ (incompressable fluid, steady flow) ...... (5)}$$

Consider the steady flow of an incompressible with streamlines and closed surfaceformed by a streamline tube shown in Fig:5.According to Eq.(5), when thespacing of the streamlines increases, the speed of the fluid must decrease. Therefore thespeed of the fluid is greater entering end-cap 1 then when it is leaving end-cap 2. Whenwe represent fluid flow by streamlines, regions in which the streamlines are widelyspaced have lower speeds than regions in which the streamlines are closely spaced.

Bernoulli's Principle:

Bernoulli's equation, one of the best-knownand widely-used results of elementary fluid dynamics.Let's again consider the case of an ideal fluid that undergoes steady flow and applyenergy methods to find an equation of state that relates pressure, density, and speed of theflow at different points in the fluid. Let's examine the case of a steady horizontal flow inas seen in the overhead view shown in Fig: 6.We represent this flow by streamlinesand a flow tube associated with the streamlines. Let's consider the motion of a fluidparticle along onestreamline passing through points *A* and *B* in Fig:6.The cross sectional area of the flow tube at point *A* is less than the cross-sectional area of the flowtube at point *B*.

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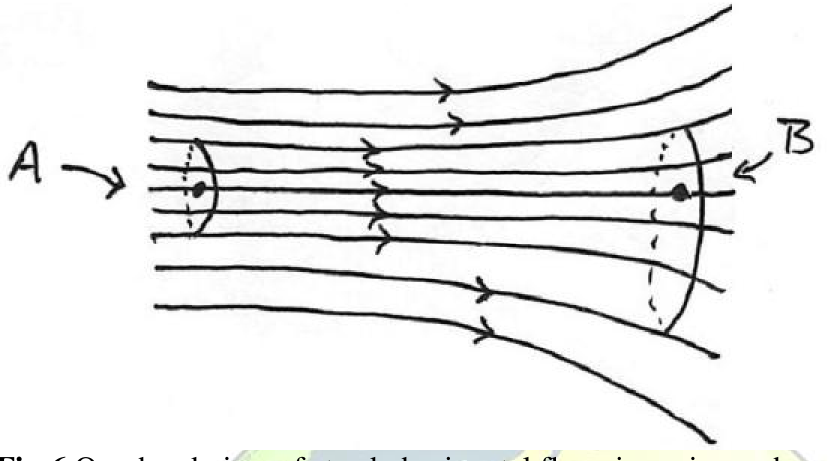

Fig:6 Overhead view of steady horizontal flow: in regions where spacing of the streamlines increases, the speed of the fluid must decrease.

According to Eq.(5), the particle located at point *A* has a greater speed than a fluid particle located at point *B*. Therefore a particle travelling along the streamline from point *A* to point *B* must decelerate. Because the streamline is horizontal, the force responsible is due to pressure differences in the fluid. Thus, for this steady horizontal flow in regions of lower speed there must be greater pressure than in regions of higher speed. Now suppose the steady flow of the ideal fluid is not horizontal, with the *y* – representingthe vertical direction. The streamlines and flow tube for this steady flow are shown in Fig:7.

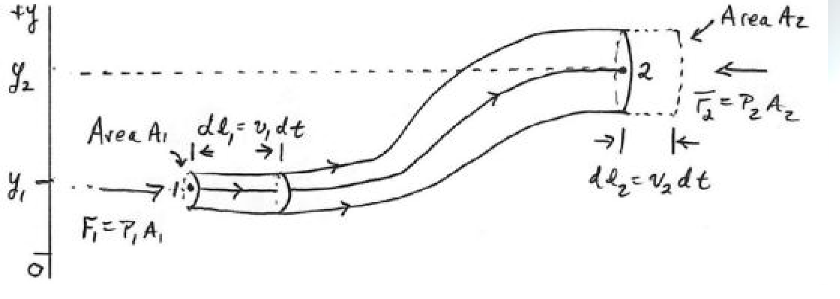

Fig:7: Non-horizontal steady flow

In order to determine the equation relating the pressure, speed and height difference ofthe tube, we shall use the work-energy theorem. We take as a system the mass containedin the flow tube shown in Fig:7.The external forces acting on our system are dueto the pressure acting at the two ends of the flow tube and the

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gravitational force.Consider a streamline passing through points 1 and 2 at opposite ends of the flow tube.Let's assume that the flow tube is narrow enough such that the velocity of the fluid is uniform on the cross-sectional areas of the tube at points 1 and 2. Point 1, denote thespeed of a fluid particle by v_1, the cross-sectional area by A_1, the fluid pressure by P_1 and the height of the centre of the cross-sectional area by y_1 .At point 2 , denote thespeed of a fluid particle by v_2 , the cross-sectional area by A_2, the fluid pressure by P_2 ,and the height of the centre of the cross-sectional area by y_2.

Consider the flow tube at time t as illustrated in Figure 28.7. At the left end of the flow,in a time interval dt, a particle at point 1 travels a distance $dl_1 = v_1 dt$. [5] proposed a principle in which another NN yield input control law was created for an under incited quad rotor UAV which uses the regular limitations of the under incited framework to create virtual control contributions to ensure the UAV tracks a craved direction. Utilizing the versatile back venturing method, every one of the six DOF are effectively followed utilizing just four control inputs while within the sight of un demonstrated flow and limited unsettling influences. Elements and speed vectors were thought to be inaccessible, along these lines a NN eyewitness was intended to recoup the limitless states. At that point, a novel NN virtual control structure which permitted the craved translational speeds to be controlled utilizing the pitch and the move of the UAV. At long last, a NN was used in the figuring of the real control inputs for the UAV dynamic framework. Utilizing Lyapunov systems, it was demonstrated that the estimation blunders of each NN, the spectator, Virtual controller, and the position, introduction, and speed following mistakes were all SGUUB while unwinding the partition Principle.

Therefore a smallvolume $dV_1 = A_1 dl_1 = A_1 v_1 dt$ of fluid is displaced at the right end of the flow tube. In asimilar fashion, at particle at point 2 , travels a distance $dl_2 = v_2 dt$.

Therefore a smallvolume of fluid $dV_2 = A_2 dl_2 = A_2 v_2 dt$ is also displaced to the right in the flow tube duringthe time interval dt. Because we are assuming the fluid is incompressible, by Eq.(5),these volume elements are equal,

$$dV = dV_1 = dV_2 \ .$$

There is a force of magnitude $F_1 = P_1 A_1$ in the direction of the flow arising from the fluidpressure at the left end of the tube acting on the mass element that enters the tube. Thework done displacing the mass element is then

$$dW_1 = F_1 dl_1 = P_1 A_1 dl_1 = P_1 dV \ ......1$$

There is also a force of magnitude $F_2 = P_2A_2$ in the direction opposing the flow arisingfrom the fluid pressure at the right end of the tube. The work done opposing thedisplacement of the mass element leaving the tube is then

$$dW_1 = -F_2dl_2 = -P_2A_2dl_2 = -P_2dV \text{ ......2}$$

Therefore the external work done by the force associated with the fluid pressure is thesum of the work done at each end of the tube

$$dW^{ext} = dW_1 + dW_2 = (P_1-P_2)dV \text{ ......3}$$

In a time intervaldt , the work done by the gravitational force is equal to

$$dW^g = -dm\, g(y_2-y_1) = -\rho dVg(y_2-y_1) \text{ ......4}$$

Because we only chose the mass in the flow tube as our system, and we assumed that thefluid was ideal (no frictional losses due to viscosity) the change in the potential energy ofthe system is

$$dU = -W^g = \rho dVg(y_2-y_1) \text{ ......5}$$

At time t , the kinetic energy of the system is the sum of the kinetic energy of the smallmass element of volume$dV = A_1dl_1$ moving with speed $v1$ and the rest of the mass in theflow tube. At time $t + dt$, the kinetic energy of the system is the sum of the kinetic energyof the small mass element of volume $dV = A_2dl_2$ moving with speed v_2 and the rest ofthe mass in the flow tube. The change in the kinetic energy of the system is due to themass elements at the two ends and therefore

$$dK = 1/2dm_2v_2^2 - 1/2dm_1v_1^2 = 1/2\rho dV(v_2^2-v_1^2)\text{.....6}$$

The work-energy theorem $dW^{ext} = dU + dK$ for system is then

$$(P_1-P_2)dV = 1/2\rho dV(v_2^2-v_1^2) + \rho g(y_2-y_1)dV\text{.....7}$$

We now divide Eq. (28.4.7) through by the volume dV and rearrange terms, yielding

$$P_1 + \rho gy_1 + 1/2\rho v_1^2 = P_2 + \rho gy_2 + 1/2\rho v_2^2 \text{ ......8}$$

Because points 1 and 2 were arbitrarily chosen, we can drop the subscripts and write Eq. (8) as

$$P + \rho gy + 1/2\rho v_2 = \text{constant (ideal fluid, steady flow)...9}$$

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Eq. (9) is known as **Bernoulli's Equation**.

Bernoulli's equation is one of the most important/useful equations in fluid mechanics. When the Bernoulli equation is combined with the continuity equation the two can be used to find velocities and pressures at points in the flow connected by a streamline.. The phenomenon that pressure decreases as velocity increases - sometimes comes in very useful in engineering. (It is on this principle that carburettor in many car engines work - Pressure reduces in a contraction allowing a small amount of fuel to enter).

Conclusion:

Streamlinesare lines which at a given instant are everywhere in the directionof the velocity(analogous to electric or magnetic field lines). In steady flow thestreamlines are independent of time, but the velocity can vary in magnitude along astreamline (as in flow through a constriction in a pipe).It should be emphasized that streamlines represent the velocity field at a specific constant of time, whereas particle paths and streaklines provide a representation ofthe velocity field over a finite period of time.

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