ISSN2394-3777 (Print)
ISSN2394-3785 (Online)
Available online atwww.ijartet.com

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. 5, Special Issue 11, April 2018

Comparative study of Piecewise Linear and Nonlinear Companding Transforms in the Reduction of Peak to Average Power Ratio

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Abstract

Peak to Average Power Ratio (PAPR) is a major drawback in OFDM modulation technique that is currently used in high speed wireless mobile communications. Companding is a well-known signal distortion scheme to reduce PAPR. Companding Transforms are computationally simple methods that depend on clipping and expanding there by transforming Rayleigh distributed amplitude of the OFDM signal into a more uniform distribution. This paper compares piecewise Linear and Nonlinear Companding Transforms in terms of efficiency in PAPR, Bit Error Rate (BER) performance, Power Spectral Density (PSD) and Computational Complexity.

Index Terms—BER, Companding, Computational Complexity, PAPR, PSD.

I.INTRODUCTION

Orthogonal Frequency Division (OFDM), a multi carrier modulation technique that is currently used in high speed wireless mobile radio standards, supports high data rates. It uses multiple overlapped subcarriers of frequencies less than the coherence bandwidth of the channel to carry data in parallel. These individual modulated subcarriers are summed together to make an OFDM signal, that exhibits a Gaussian distribution and gains the ability to overcome the frequency selectivity of a wide band channel. But it results in a large crest factor because Of the addition of large number of sub carriers that occur in phase. The high crest factor or large Fluctuations in the envelope are a huge setback for OFDM.Large envelope variations indicated by PeakTo Average Power Ratio demands a highly efficient High Power Amplifier with a large back-off, up-convertors with high linearity and Digital to Analog convertors with large dynamic range. This problem is more pronounced in the case of mobile user terminals where there is a limitation to the battery power.

Further there exists a trade-off between PAPR, Bit Error Rate and Power Spectral density; the reduction in one parameter degrades the other. Any PAPR reduction scheme must therefore be wisely selected. Computational Complexity is another parameter of concern, where the implementation of complex techniques require additional power, also if the scheme is more computational intensive takes more time and introduces latency in the network, and becomes spectrally inefficient.

Clipping, a nonlinear simple and straight forward technique that deliberately clips large signal amplitudes introduces specific levels of clipping noise. This can be considered impulsive in nature rather than continual Gaussian noise [1]. It also causes in band distortion, out of band radiation and degrades the BER performance. Instead the Companding schemes softly limits the signal amplitudes around a selected threshold also

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maintaining average signal power if the parameters are properly selected.

In [2] authors proposed a companding transform which reduces PAPR keeping the peak of the OFDM signal unaltered and increasing the average power, which is not so optimistic in practical scenario. In [3] authors propose a general companding transform, and analyses the four specific case studies namely linear symmetrical (LST), linear non symmetrical (LNST), nonlinear symmetrical (NLST), nonlinear non symmetrical (NLNST) transforms. Among these LNST is proved to be the best where performance gain is derived by introducing an inflexion point there by gaining more degrees of freedom. In [4], authors introduced a nonlinear companding scheme that improves system performance with low out of band distortion, further analyzing the decompanding operation, shows that decompanding amplifies the channel noise.

In [4] authors compare the simple linear and nonlinear companding transforms and proves the superiority of linear transforms, also derives relationship between companding parameters to keep average power unchanged. In [5] authors adopt inflexion point in nonlinear companding scheme to induce more flexibility in selection of companding parameters. In [6] authors, introduce a piecewise linear companding transform with mitigation of companding distortion.

The remainder of this paper is organized as follows. Section II presents the system model and theoretical analysis of PAPR. Section III describes the piecewise linear and nonlinear companding transforms. Results are given section IV. Section V presents the conclusion of the study of both the transforms.

II. SYSTEM MODEL AND THEORETICAL ANALYSIS

Description of OFDM system

Figure 1 shows the block diagram of an OFDM system, withCompanding transform block. Summation of multiple modulated orthogonal subcarriers is physically realized using an IFFT. The sampled version of the OFDM signal is given as

$$x(n) = \frac{1}{\sqrt{N}}\sum_{k=0}^{NM-1} X(k)e^{\frac{j2\pi kn}{NM}}, 0 \le n < NM-1, (1)$$

Where X (k)=[X(0), X(1),…………..X(N)] are *i.i.ds* that represent constellation points of M-QAM scheme. N is the number of sub carriers. M is the oversampling factor, PAPR of continuous OFDM can be approximated if the discrete signal is M times oversampled, which can be done by padding N(M-1) zeros at the center of X(k). Central limit theory approximates the signal has asymptotically Gaussian distribution when the number of sub carriers are large enough. But the amplitude of this complex OFDM signal **|x (k)|** assumes a Rayleigh distribution with the probability density function as

$$f_{|x(k)|}(x) = \frac{2x}{\sigma_x^2}e^{-\frac{x^2}{\sigma_x^2}}, \quad x \ge 0, \quad (2)$$

Where $\sigma_x^2$is the variance of x(n), variance of any statistical signal is expectation of square of the of absolute value of the signal. Therefore power of the transmitted signal is given by $\sigma_x^2 = E(|X(k)|^2)$

The Peak to Average Power Ratio is given as

$$PAPR = 10log_{10}\frac{max\,(|x(n)|^2)}{E(|x(n)|^2)}(3)$$

Theoretical analysis of Companding, compressing and expanding around a threshold is an additional manipulation on OFDM signals inducing distortion that degrades the BER performance. The effect of distortion on BER is derived as follows.

Consider z (n) is companded version of the input x(n)

$$z(n) = x(n)+c(n)\ (4).$$

c(n) is the companding distortion signal with same phase as x(n), Then the power of companded signal is given as

$$\sigma_z^2 = \sigma_x^2 + 2E\left(c(n)^*x(n)\right) + \sigma_c^2(5)$$

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To maintain average power constant, $\sigma_z^2 = \sigma_x^2$ power in the compressed part of the signal must be equal to power in the expanded part

$$-2E\big(c(n)^* x(n)\big) = \sigma_c^2 \quad (6)$$

According to Bussgang theorem, companded signal can again be divided into a scaled input and uncorrelated distortion part d(n)

$$z(n) = \alpha\, x(n) + d(n) \quad (7)$$

where attenuation factor α is the ratio of companded signal power to the original OFDM signal power and companded signal z(n) is transformed version of the x(n) therefore its power can be computed using $z^*(n)x(n)$. Substituting (4) in (8), we get,

$$\alpha = \frac{\sigma_z^2}{\sigma_x^2} = \frac{E(|z(n)^* x(n)|}{E(|x(n)|)} = 1 - \frac{\sigma_c^2}{2\sigma_x^2} \quad (8)$$

The companded signal after passing the AWGN channel can be denoted by p(n)

$$p(n) = z(n) + w(n) = \alpha x(n) + d(n) + w(n), \quad (9)$$

Where, w(n) is the additive Gaussian noise, finally decompanded version of received signal is obtained by

$$x'(n) = \frac{p(n) - d(n)}{\propto} \quad (10)$$

Then the Signal to noise ratio at the receiver is given by

$$SNR = \frac{|\alpha|^2 \sigma_x^2}{\sigma_w^2} = \left(1 - \frac{\sigma_c^2}{2\sigma_x^2}\right)\frac{\sigma_x^2}{\sigma_w^2} \quad (11)$$

Thus BER performance is improved when attenuation factor is decreased thereby decreasing companding distortion

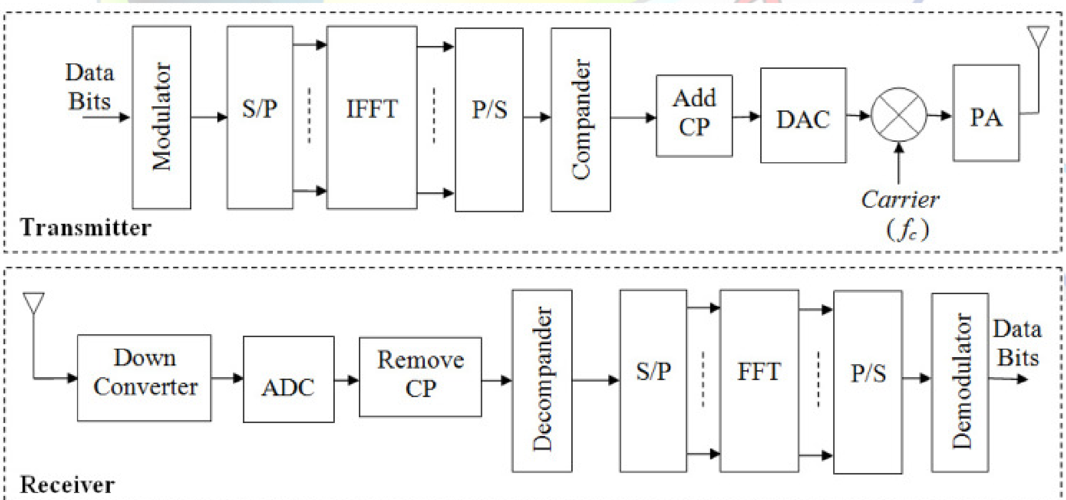

Fig1. Block diagram of typical OFDM system with companding block

Piecewise companding

Figure 2 shows the Rayleigh distributed amplitude of OFDM signal. Power increment during expansion depends not only on the sample amplitude but also on the probability distribution, Section I shows the samples with smaller amplitudes Section III indicates samples with large amplitudes and finally Section III represents the samples with amplitudes that will get into the saturation region of High Power Amplifier.

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Remembering, the Power of any sample is the square of its amplitude.

To compensate for the power loss of the samples in Section III that are attenuated during compression operation can be done in two ways. One method treats all the samples that lie below a threshold or companding peak in a unified manner, by expanding the power with a similar scale. The other is piecewise method, It compands samples distributed in different region with different scales. Even with small increments in the power of samples in Section II, as shown in fig 2, average power can be maintained constant. Because larger amplitude signals, provide large power even with a fewer samples that fall in the region of Section III, to compensate for the power loss in clipped part. Thus companding distortion can be reduced

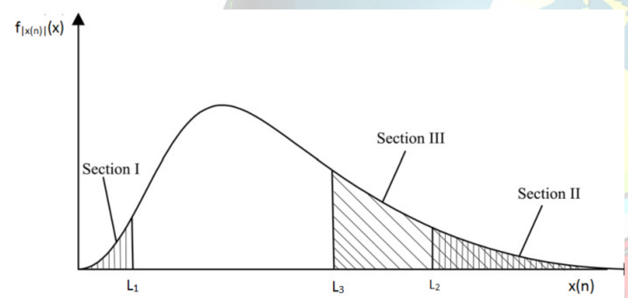


Fig. 2 Probability density function of |x(n)|

III. Piecewise Linear and Nonlinear Companding Transforms

Piecewise Linear companding (PLC)

In piecewise linear companding technique, companded signal is a linear transformation of the input signal where the signal amplitudes over the peak power A_c are clipped. To maintain average power, amplitudes near the peak power are expanded with a linear scale.

Figure 3 shows PLC scheme. Where companding transform function given as

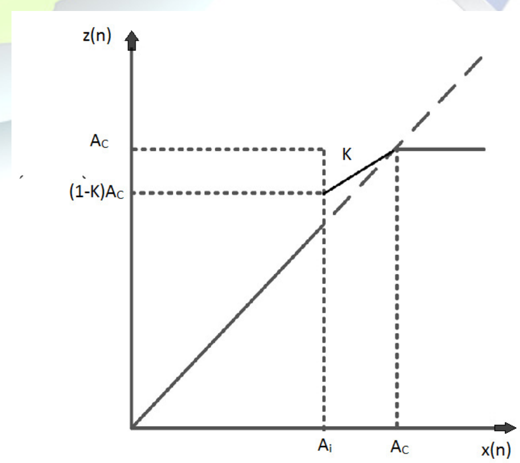
$$P_L(x) = \begin{cases} x \quad , & |x| \le A_i, \\ kx + (1-k)A_c, & A_i < |x| \le A_c, \\ sign(x)A_c \,, & |x| > A_c, \end{cases} \quad (12)$$

To revert the companding operation at the receiver, the inverse of the companding transform is used as decompanding transform. Decompanding transform is therefore given by

$$P_L^{-1}(x) = \begin{cases} x \quad , & |x| \le A_i, \\ \big(x-(1-k)\big)Ac/k \,, & (1-k)A_i < |x| \le A_c \\ sign(x)A_c, & |x| > A_c, \end{cases} \quad (13)$$

In equations (12) & (13), k is the scaling factor, A_c is the companding peak, $A_i$is the inflexion point. Broadly speaking, The PLC scheme just disintegrates into clipping and amplifying operations. Piecewise linear scheme compands amplitudes of the signals that lie above A_i only there by keeping attenuation factor low as explained in Section I.

The inflexion point A_i refers to the transition point in the companding and decompanding functions. The parameters A_i, A_c, k can be derived intuitively. Maintaining constant average signal power before and after companding further reduces distortion. So by equating average signal powers as shown in equation (14), A_i and k can be derived for preset PAPR. Where $A_c = \sigma_x 10^{preset/20}$. To keep the average signal power constant, k has to be a positive real number less than 1



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Vol. 5, Special Issue 11, April 2018

Fig3. Profile of Piecewise Linear Companding Transform

$$\int_{Ai}^{Ac} (\boldsymbol{kx} + (\mathbf{1}-\boldsymbol{k})\boldsymbol{Ac})^2 \boldsymbol{f}_{|x(n)|}(\boldsymbol{x})\boldsymbol{dx} + \int_{Ac}^{\infty} \boldsymbol{Ac}^2 \boldsymbol{f}_{|x(n)|}\boldsymbol{dx} = \int_{Ai}^{\infty} \boldsymbol{x}^2 \boldsymbol{f}_{|x(n)|}\boldsymbol{dx} \quad (14)$$

PLC avoids unnecessary compression and expands large amplitude signals with small increments.

Piecewise Nonlinear companding (PNC)

In piecewise nonlinear companding Rayleigh distribution of amplitude of original OFDM signal is approximated using a strictly increasing function whose first derivative does not change.

Consider z(n) is the companded signal whose amplitude is raised to the power β has a uniform distribution in the interval $|z(n)|^{\beta} \in [Ai, Ac]$. The pdf of such signal the given by

$$f_{|z(n)|} = \begin{cases} \frac{2z}{\sigma_z^2} e^{-(z^2/\sigma_z^2)}, & z \in [0, Ai) \\ d\frac{e^{Ai^2/\sigma_z^2}}{Ac^d - Ai^d}, & z \in [Ai, Ac] \end{cases} \quad (15)$$

Referring to fig.3, till the inflexion point Ai signal amplitudes are unaltered therefore distribution remains the same, signals whose amplitudes exceed a companding peak Ac are softly clipped using a monotonic function simultaneously expanding the signals that lie in the interval of inflexion point to companding peak .

Increasing Monotonic function h(x) that transforms the distribution of companded signal as defined above must therefore satisfy equation (16).

$$\int_0^x f_{|x(n)|}(x)dx = \int_0^{h(x)} f_{|z(n)|}(z)dz \quad (16)$$

Here, h(x) is the required companding function, to avoid sudden change in the amplitude additional factor, $C = A_c^d$, is further added to minimize distortion

Figure 4 shows the PNC scheme. Where Companding transform is given as

$$H(x) = \begin{cases} x, & ,|x| < A_i, \\ sgn(x)((A_i^\beta - A_c^\beta)e^{(|x(n)|^2 - A_i^2)/\sigma_x^2} + A_c^d)^{\left(\frac{1}{\beta}\right)}, & |x| \geq A_i, \end{cases} \quad (16)$$

While, Decompanding Transform is given as

$$h^{-1}(x) = \begin{cases} x, & |x| < Ai \\ sgn(x)\sqrt{A_i^2 - \sigma_x^2 \ln\left(\frac{|x|^\beta - A_c^\beta}{A_i^\beta - A_c^\beta}\right)}, & |x| \geq Ai \end{cases} \quad (17)$$

To derive A_i and β, for a given preset A_c that determines the theoretical PAPR signal powers are equated.

$$\int_0^\infty x^2 f_{|x(n)|}(x) = \int_0^{A_c} z^2 f_{|z(n)|}(z)dz, \quad (18)$$

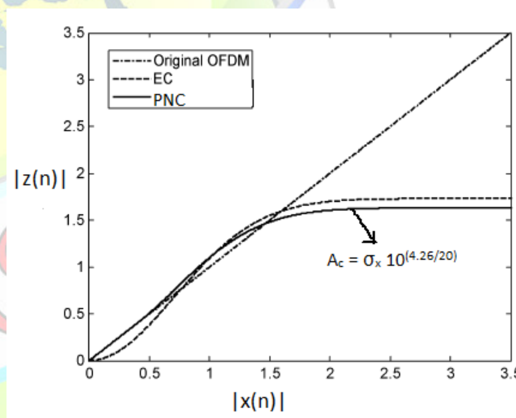

Fig.4 Profile of Piecewise Nonlinear Companding

Comparisional Analysis

The major difference that can be observed in PLC and PNC is the role of scale factor or the attenuation factor (∝).

$$\alpha = \frac{1}{\sigma_x^2}\int_0^\infty xh(x)f_{|x(n)|}(x)dx, \quad (19)$$

Considering the result in equation (11),

In PLC, the scale factor is reduced by mitigating companding distortion to improve the system

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performance But in PNC, based on relation between the scale factor and the companding function, to improve the system performance 'α' is made large by choosing proper parameters. Another difference is in decompanding at the receiver where, Decompanding operation amplifies the channel noise w(n) to w(n)/∝, as proved in [5].

IV .PERFORMANCE EVALUATION

In order to evaluate the efficiency in PAPR reduction of these companding transforms and to compare their effect on other parameters of the system, a MATLAB simulation is performed. We consider a baseband OFDM with WiMAXstandards. According to IEEE 802.16 standards, the Number of subcarriers are 256, of which 192 are used for data, 8 for pilots, 57 for guard bands, one for DC. An oversampling factor of 4 is used to derive the correct estimate of PAPR. We used, 4QAM baseband modulation scheme and AWGN channel is implemented.

To find BER, PSD, PAPR, Monte-Carlo estimations are performed. Both OFDMA and single carrier frequency division multiplexing (SCFDMA) modulation schemes are implemented. LTE uses OFDMA in its downlink technology, and SCFDMA in its uplink technology to reduce PAPR further. SCFDMA is a single carrier technique that retains the essential frequency diversity feature of OFDM. To implement SCFDM in frequency domain, additional DFT block are added before sub carrier mapping, that cancel out the effect of multiple carriers virtually and manipulates it to be a single carrier signal. In the simulations SCFDM is represented as 'dft precoded'.

For the sake of fair comparison PAPR preset value is taken to be 4.26 for both the companding schemes. Where both the schemes perform reasonably well at the selected preset.

Figure.5 depicts the Bit Error Rate performance of PLC and PNC in different cases

As mentioned in Section III (B), for PNC simulations without decompanding are also done. Here nd represents 'no decompanding', while wd represents 'with decompanding'. At BER level of 10^{-4}, with PAPR preset of 4.26 dB, the minimum required Eb/N0 for PLC and PNC are 8.9 and 9 reducing PAPR to 4.9 and 4.5 simultaneously. in the case of SCFDM or the DFT precoded PLC and PNC required Eb/N0 are 9.9 and 8.7 respectively. DFT precoded PLC has performance error floor at high Eb/N0 because of the discontinuity of the companded signals. The numerical results of all the specific cases are summarized in Table I

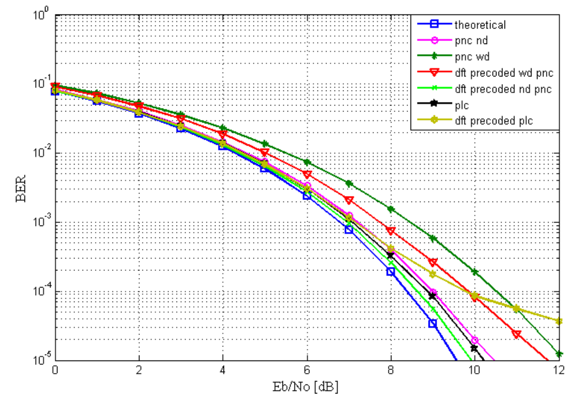

Fig.5 BER performance of original OFDM signal and companded signals over AWGN channel with 4-QAM modulation

Table I

Performance comparison

Performance comparison of PAPR reduction and BER performance for PLC and PNC		
Companding Schemes	At BER = 10^{-4} E_b/N_0 [dB]	At CCDF $=10^{-3}$ PAPR (dB)
Original OFDM	8.4	11.8

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precoded OFDM	8.4	8
PLC	8.9	4.9
precoded PLC	9.9	4.4
PNC	10.5	4.5
PNC without Decompanding	9	4.5
DFT precoded PNC (DPNC)	9.9	3.9
DPNC without Decompanding	8.7	3.9

Figure.6 shows the complementary cumulative distribution of Peak to Average Power ratio, for PLC and PNC in different cases. DFT preceded PNC performs well limiting the peak to 3.9 at a CCDF level of 10^{-3}. While PLC limits peak to 4.9 without degrading BER much.

Figure.7 represents the power spectral density plots of PLC and PNC and DFT preceded PLC and PNC. Because of the introduction of inflexion points in the companding transforms PSD degrades more. But due to the compensation introduced in PNC, out of band emission declines more rapidly in PNC.

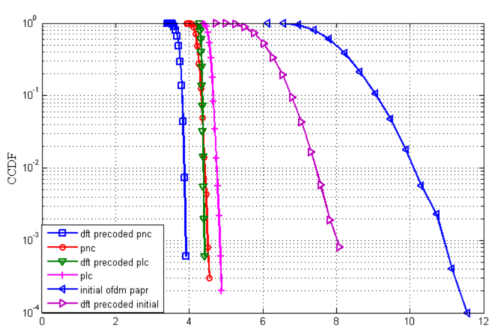

Fig.6 CCDF's of original OFDM and companded signals.

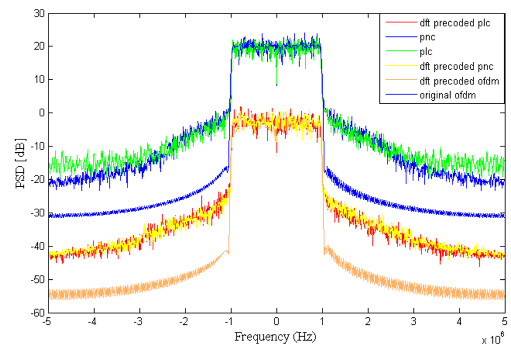

Fig.7 Power Spectrums of original and companded signals

Finally, comparing the computational efforts required for different companding schemes which is measured in terms Flops/Sample. Flops are floating point operations required to implement the operation in MATLAB. PLC proves to be the very less complex requiring only 1.3NM flops where PNC requires 6.7NM flops. N is number of subcarriers and M is the oversampling factor. Specifically flop count does not include signal amplitude computations here.

V. CONCLUSION

This paper reviews both Piecewise Linear and Nonlinear Companding Transforms. It concludes that BER performance wise Piecewise Nonlinear companding without Decompanding is better compared to Piecewise Linear companding. But computational complexity is less for Linear Transform. Clear analysis of Inflexion point for both schemes is presented. The difference between both schemes is analyzed in Section III.

ACKNOWLEDGMENT

This article is purely a comparativestudy of the papers [5] and [6] in the references. Simulation for all the scenarios is done specifically. Author acknowledges and thanks the writers of papers [5] and [6] for the insightful ideas they presented.

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