ISSN 2394-3777 (Print)
ISSN 2394-3785 (Online)
Available online at www.ijartet.com

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. 5, Special Issue 12, April 2018

Total ν-Strong Domination in Intuitionistic Fuzzy Graph

R. Buvaneswari [1], K.Jeyadurga [2]

Assistant Proffessor, Department of Mathematics, KG College of Arts and Science, Coimbatore, India [1]
Research Scholar, Department of Mathematics, KG College of Arts and Science, Coimbatore, India [2]
buvaneswari.r@kgcas.com[1], jayamahesh245@gmail.com[2]

***Abstract:*In this paper, ν-strong(weak) domination and total ν–strong (weak) domination in intutionistic fuzzy graph are defined. The properties of the total ν–strong (weak) domination in intutionistic fuzzy graph are analyzed.**
Keywords: Domination intutionistic fuzzy graph, Strong (weak) intutionistic fuzzy graph, Total strong (weak) in intutionistic fuzzy graph, Total ν-strong domination in intutionistic fuzzy graph.

I.Introduction

In 2016, strong domination intutionistic fuzzy graph was discussed by C. V. R. Harinarayanan, et al. [1]. In [2], the authors introduced intuitionistic fuzzy graph and studied some properties. The author [3] defined constant intuitionistic fuzzy graph and derived important results in constant intuitionistic fuzzy graph. Karunambigai et al. [4,5] classified the arcs in intuitionistic fuzzy graph based on the strength of connectedness between two vertices. Also, the vertices are classified as strong and super strong depending upon the classification of arcs by the same authors. In [6], A. Nagoor Gani and S. Shajitha Begum defined some properties of degree, order and size in intuitionistic fuzzy graph. Some domination parameters in intuitionistic fuzzy graph and their properties are discussed by N. Vinoth Kumar, and G. Geetha Ramani[7]. In this paper, the authors defined the total ν–strong domination in intutionistic fuzzy graph and studied the domination properties.

II.Preliminaries

Definition 2.1[2]

Let G=(V,E) be an intutionistic fuzzy graph, such that

(i) $V=\{v_1,v_2.....v_n\}$ such that $\mu_i : V \to [0,1]$, $\nu_i : V \to [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \le \mu_i(v_i)+\nu_i(v_i) \le 1$ for every $v_i \in V$, (i=1,2,......n).

(ii) $E \subseteq V \times V$ where $\mu_{ij} : V \times V \to [0,1]$ and $\nu_{ij} : V \times V \to [0,1]$ are such that $\mu_{ij}(v_i,v_j) \le \mu_i(v_i) \wedge \mu_i(v_j)$ and $\nu_{ij}(v_i,v_j) \le \nu_i(v_i) \wedge \nu_i(v_j)$ respectively and $0 \le \mu_{ij}(v_i,v_j)+\nu_{ij}(v_i,v_j) \le 1$

Definition 2.2[1]

Let G = (V, E), be an intutionistic fuzzy graph. Then the cardinality of G is defined to be

$$|G| = \left| \sum_{v_i \in V} \frac{1+\mu_1(v_i)-\nu_1(v_i)}{2} + \sum_{v_i \in V} \frac{1+\mu_2(e_{ij})-\nu_2(e_{ij})}{2} \right|$$

The vertex cardinality is defined to be

$$|V| = \sum_{v_i \in V} \frac{1+\mu_1(v_i)-\nu_1(v_i)}{2}$$

The edge cardinality is defined to be

$$|E| = \sum_{e_{ij} \in V} \frac{1+\mu_2(e_{ij})-\nu_2(e_{ij})}{2}$$

Definition 2.3[2]

Let G=(V,E), be an intutionistic fuzzy graph. Then the complement of an intutionistic fuzzy graph is denoted by $G = (\overline{V}, \overline{E})$. If it satifisfies the conditions,

(i) $\overline{V} = v$

(ii) $\overline{\mu_{1i}} = \mu_{1i}$ and $\overline{\nu_{1i}} = \nu_{1i}$ for all i=1,2,.....n

(iii) $\overline{\mu_{2ij}} = \min(\mu_{1i}, \mu_{1j}) - \mu_{2ij}$ and

$\overline{\nu_{2ij}} = \min(\nu_{1i}, \nu_{1j}) - \nu_{2ij}$ for all i,j=1,2,...n

Definition 2.4[3]

Let G=(V,E) be an intutionistic fuzzy graph. Then the degree of a vertex V is defined by $d(v) = (d_\mu(v), d_\nu(v))$ where $d_\mu(v) = \sum_{i \neq j} \mu_2(\mu_{ij})$ and $d_\nu(v) = \sum_{i \neq j} \nu_2(\nu_{ij})$

Definition 2.5[2]

The effective degree of a vertex V in intutionistic fuzzy graph, G = (V,E) is defined to be the sum of the strong edge incident at V. It is denoted by $\delta_E(G)$ and $\Delta_E(G)$.

The minimum degree of G is $\delta_E(G) = \min(d_E(v)/v \in V)$.

The maximum degree of G is $\Delta_E(G) = \max(d_E(v)/v \in V)$

Two vertices v_i and v_j are said to be neighbourhood in intutionistic fuzzy graph there is a strong arc between v_i and v_j.

Definition 2.6[2]

An intutionistic fuzzy graph $H' = (V', E')$ is said to be an intutionistic fuzzy sub graph of G = (V, E) if $V' \subseteq V$ and $E' \subseteq E$. That is $\mu'_{1i} \leq \mu_{1i}$; $\nu'_{1i} \leq \nu_{1i}$ and $\mu'_{2ij} \leq \mu_{2ij}$; $\nu'_{2ij} \leq \nu_{2ij}$ for every i,j=1,2,.....n

Definition 2.7[2]

A path in an intutionistic fuzzy graph is a sequence of distinct vertices $v_1, v_2, \ldots. v_n$ such that either one of the following condition have to be satisfied.

$\mu_2(\mu_{ij}) > 0, \nu_2(\nu_{ij}) > 0$ for some i and j

$\mu_2(\mu_{ij}) = 0, \nu_2(\nu_{ij}) > 0$ for some i and j

$\mu_2(\mu_{ij}) > 0, \nu_2(\nu_{ij}) = 0$ for some i and j

Definition 2.8[1]

Let G = (V,E) be an intutionistic fuzzy graph. Let $u, v \in V$, we say that u dominates v in G if there exists a strong arc between them. A subset $D \subseteq V$ is said to be dominating set in G if for every $v \in V - D$, there exists u dominates v.

Definition 2.9[1]

A dominating set D of intutionistic fuzzy graph is said to be minimal dominating set if no proper subset of S is a dominating set. Minimum cardinality among all minimal dominating set is called the intutionistic fuzzy domination number, and is denoted by $\nu_{if}(G)$.

Definition 2.10[1]

Let u and v be any two vertices of an intutionistic fuzzy graph of G. The vertex u strongly dominates v (u weakly dominates v) if

(i) strong arc between u and v

(ii) $d_N(u) \geq d_N(v)$

A strong (weak) dominating set S of an IFG is said to be minimal strong (weak) dominating set if no proper subset of S is strong (weak) dominating set of G. The minimum cardinality among all minimal strong (weak) dominating set is called strong (weak) intutionistic fuzzy domination number of G, and is denoted by $\nu_{sif}(G)$ and $\nu_{wif}(G)$.

Definition 2.11[1]

The set S is said to be total dominating set if for every vertex $v \in \nu(G)$, V dominates to atleast one vertex of S.

Definition 2.12[2]

An intutionistic fuzzy graph G=(V,E), is said to be strong intutionistic fuzzy graph if $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$ and $\nu_{2ij} = \min(\nu_{1i}, \nu_{1j})$ for all $\nu_i, \nu_j \in E$

Definition 2.13[1]

Let u and v be any two vertices of a intutionistic fuzzy graph G. Then u strongly dominates v (v weakly dominates u) if (i) $\nu(u,v) \geq \sigma(u) \wedge \sigma(v)$ (ii) $d_N(u) \geq d_N(v)$ (iii) for every vertex $v \in \nu(G)$; v dominates to atleast one vertex of S.

Definition 2.14[1]

i. An intutionistic fuzzy graph is of the form G=(V,E)where $V = \{v_1, v_2, \ldots. v_n\}$ such that $\mu_i : V \to [0,1], \nu_i : V \to [0,1]$ denotes the degree of the membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \mu_i(v_i) + \nu_i(v_i) \leq 1$ for every $v_i \in V$

ii. $E \subseteq V \times V$ where $\mu_{ij} : V \times V \to [0,1], \nu_{ij} : V \times V \to [0,1]$ and, $\mu_{ij}(v_i, v_j) \leq \min(\mu_i(v_i), \mu_j(v_j))$,

ISSN 2394-3777 (Print)
ISSN 2394-3785 (Online)
Available online at www.ijartet.com

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. 5, Special Issue 12, April 2018

$\nu_{ij}(v_i,v_j) \le \min(\nu_i(v_i),\nu_j(v_j))$ and $0 \le \mu_{ij}(v_i,v_j)+\nu_{ij}(v_i,v_j) \le 1$ for every $(v_i,v_j) \in E$

iii. $\nu(u,v) \ge \sigma(u) \wedge \sigma(v)$

iv. $d_N(u) \ge d_N(v)$ and

v. for every vertex $v \in V(G)$; v dominates to atleast one vertex of S.

Definition 2.15[1]

The vertex cardinality of an intutionistic fuzzy graph is called the order of G and it is denoted by O(G). Order of G is defined by $O(G)=(O_\mu(G),O_\nu(G))$ where $O_\mu(G)=\sum_{i=1}^{n}\mu_i$ and $O_\nu(G)=\sum_{i=1}^{n}\nu_i$

Definition 2.16[1]

The edge cardinality of an intutionistic fuzzy graph is called the size of G and it is denoted by S(G). Size of G is defined by $S(G)=(S_\mu(G),S_\nu(G))$ where $S_\mu(G)=\sum_{i\ne j}\mu_{ij}$ and $S_\nu(G)=\sum_{i\ne j}\nu_{ij}$

Definition 2.17[5]

An intutionistic fuzzy graph, G=(V,E) is said to be semi- ν strong IFG if $\nu_{ij}=\min(\nu_i,\nu_j)$ for every i and j

Definition 2.19[1]

Let u and v be any two vertices of a intutionistic fuzzy graph G. Then u strongly dominates v (v weakly dominates u) if

(i) strong arc between u and v

(ii) $d_N(u) \ge d_N(v)$

(iii) $\nu_{ij}=\min(\nu_{1i},\nu_{1j})$

III.Total ν -strong domination in intuitionistic fuzzy graph

Theorem : 3.1

Let G=(V,E) be an intutionistic fuzzy graph. Assume that D be an minimal total ν strong intutionistic fuzzy dominating set of G. Then for each $v \in D$ if and only if the following condition satisfies

(i) No vertex in D ν -strongly dominates v

(ii) There exist $u \in V-D$ such that v is the only vertex in D which ν strongly dominates u.

Proof

Necessary:

This is necessary to prove no vertex in D, ν -strongly dominates u. Assume that D is a minimal total ν strong intutionistic fuzzy dominating set of G. Then for every vertex $v \in D$, $D-u$ is not a total ν strong dominating set and hence there exist $u \in V-(D-\{v\})$, which is not ν strongly dominated by any vertex in $D-\{v\}$. If $u=v$, we get, u is not ν strongly dominated by vertex in D. If $v \ne u$, u is not ν strongly dominated by $D-\{u\}$, but u is ν strongly dominated by D, then the vertex u is ν strongly dominated by a vertex v in D.

Sufficient:

It is sufficient. Conversely assume that D is a total ν strong dominating set and for each vertex $v \in D$, one of the conditions holds.

i. Suppose D is not a minimal total ν strong dominating set, then there exists a vertex $v \in D$, $D-\{v\}$ is a Total ν strong dominating set. Hence v is strong dominated by atleast one vertex in $D-\{v\}$, the first condition does not hold.

ii. If $D-\{v\}$ is a total ν strong every vertex in $v-D$ is ν strongly dominated by at least one vertex in $D-\{v\}$, the second condition does not hold.

which is contradiction to that atleast one of these conditions hold. So D is a minimal total ν strong dominating set.

Theorem : 3.2

Let G=(V,E) be an intutionistic fuzzy graph. Assume that D be an minimal total ν -strong (weak) intuitionistic fuzzy dominating set G. Then for each $v \in D$ if and only if the following condition satisfies.

(i) No vertex in D ν weakly dominates v

(ii) There exist $u \in V-D$ such that v is the only vertex in D which ν weakly dominates u.

ISSN 2394-3777 (Print)
ISSN 2394-3785 (Online)
Available online at www.ijartet.com

International Journal of Advanced Research Trends in Engineering and Technology (IJARTET)
Vol. 5, Special Issue 12, April 2018

Theorem : 3.3

The order of $O(G)$ for an intutionistic fuzzy graph

(i) $\nu_{sif}(G) \le \nu_{tvsif}(G) \le O(G) - \Delta_N(G) \le O(G) - \Delta_E(G)$

(ii) $\nu_{wif}(G) \le \nu_{twif}(G) \le O(G) - \delta_N(G) \le O(G) - \delta_E(G)$

Proof:

Since every total ν strong intutionistic fuzzy dominating set (total ν weak intutionistic fuzzy dominating set) is intutionistic fuzzy dominating set of intutionistic fuzzy graph G,

$\nu_{sif}(G) \le \nu_{tvsif}(G)$ and $\nu_{wif}(G) \le \nu_{twif}(G)$.

Let $u, v \in V$, if $d_N(u) = \Delta_N(G)$ and $d_N(v) = \delta_N(G)$. Then $V - N(u)$ is a total ν strong intutionistic fuzzy dominating set but not minimal and $V - N(v)$ is a total ν weak intutionistic fuzzy dominating set but not minimal

Therefore $\nu_{tvsif}(G) \le |V - N(u)|_{sif}$

$$\text{(ie)} \quad |V - N(u)|_{sif} = |V| - |N(u)|$$
$$= O(G) - d_N(u)$$
$$= O(G) - \Delta_N(G)$$
$$\Rightarrow \nu_{tvsif}(G) \le O(G) - \Delta_N(G)$$

And $\nu_{twif}(G) \le |V - N(v)|_{wif}$

$$\text{(ie)} \quad |V - N(v)|_{wif} = |V| - |N(v)|$$
$$= O(G) - d_N(v)$$
$$= O(G) - \delta_N(G)$$
$$\Rightarrow \nu_{tvsif}(G) \le O(G) - \delta_N(G)$$

Further, since

$\Delta_E(G) \le \Delta_N(G)$ and $\delta_E(G) \le \delta_N(G)$

$$\Rightarrow \nu_{sif}(G) \le \nu_{tvsif}(G) \le O(G) - \Delta_N(G) \le O(G) - \Delta_E(G)$$

$$\Rightarrow \nu_{wif}(G) \le \nu_{twif}(G) \le O(G) - \delta_N(G) \le O(G) - \delta_E(G)$$

IV.Conclusion

In this paper, the total strong (weak) domination intuitionistic fuzzy graph defined and studied the properties in domination. The authors are very interested in developing intuitionistic fuzzy graphs. The authors therefore proposed to continue their work in the various field of application.

Reference

[1] C. V. R. Harinarayanan and P. Muthuraj and P. J. Jayalakshmi, "Total strong (weak) domination intutionistic fuzzy graph", *Advances in theoretical and applied mathematics*, 11 (2016), 203-212.

[2] M. G. Karunambigai and R. Parvathi, "Intuitionisitc Fuzzy Graph"s, Proceedings of 9th Fuzzy Days International Conference Intelligence, Advances in soft computing: Computatinal intelligence; Theory and Applications, *Springer-Verlag*, 20 (2006), 139-150

[3] M. G. Karunambigai and R. Parvathi and R. Buvaneswari, "Constant intutionistic fuzzy graphs", *Notes on Intuitionistic Fuzzy system,* 17 (2011), 1, 37-47.

[4] M. G. Karunambigai and R. Parvathi and R. Buvaneswari, "Arcs in intutionistic fuzzy graph", *Notes on Intuitionistic Fuzzy system,* 17 (2011).

[5] M. G. Karunambigai and Muhammad Akram and R. Buvaneswari, "Strong and super strong vertices in intutionistic fuzzy graphs", *Journals on intuitionistic fuzzy system,* 30 (2016), 671-678.

[6] Nagoor Gani and S. Shajitha Begum, "Degree, order and size in intutionistic fuzzy graph", *International Jouranal of Algorithm, Computing and Mathematics*, (3) 3 (2010)

[7] N. Vinoth Kumar, and G. Geetha Ramani, "Some domination parameters of the intutionistic fuzzy graph and its properties", *International Journal of Computational Sciences and Mathematics*, P. 62-65.